LANGE AND LAWS, KINDS, AND COUNTERFACTUALS

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Abstract

In this paper I examine and question Marc Lange's account of laws, and his claim that the law delineating the range of natural kinds of fundamental particle has a lesser grade of necessity that the laws connecting the fundamental properties of those kinds with their derived properties.

1 Lange on laws

Regularity theorists about laws face the following problem. Many regularities are true, but only some of them correspond to laws. Consider

(S1) all bits of copper conduct electricity;
(S2) all lumps of gold have a volume of less than a cubic mile.

These two generalizations are both true. However, of:

(N1) it is a law that all bits of copper conduct electricity;
(N2) it is a law that all lumps of gold have a volume of less than a cubic mile.

only the first is true. The trick for the regularity theorist is to pick from among the regularities those that are like (S1), for which the addition of the *nomic* operator 'it is a law that ...' is truth preserving, while excluding those like (S2), for which that operator yields a falsehood.

Marc Lange is no regularity theorist. But his starting question is very similar to that posed for the regularity theorist. Consider all the true propositions that are not facts about which laws there are—propositions that can be expressed without using vocabulary such as 'law', 'nomological' and the like. Call this set 'Σ'. Propositions such as (S1) and (S2) are in Σ because they are true and don't concerns laws. (N1) is excluded because although true it concerns a law. (N2) is doubly excluded because it concerns an alleged law and is false. Σ is the set of what Lange call the 'sub-nomic' facts. They are 'sub-nomic' because they don't concern which laws they are; they are not expressed using nomic vocabulary. Some of the facts in Σ are those like (S1) for which the nomic operator is truth preserving. These are, of course, the *laws*.[1] I'll use 'accident' as the term for the remaining members of Σ that are like (S2), for which the nomic operator takes us from a truth to a falsehood.

How do we choose the laws in Σ from the accidents? Lange's approach is to exploit their superior stability under counterfactual suppositions. Consider:

(G1) Were Bill Gates to want some non-conducting copper, all bits of copper would still conduct electricity;

(G2) Were Bill Gates to want wants a lump of gold with a volume greater than a cubic mile, all lumps of gold would still have a volume of less than a cubic mile.

These are counterfactual (or, better, subjunctive) conditionals of a mildly unusual sort, where although the antecedent is counter-to-fact (i.e. false), their consequents are true. But they do not require and special kind of treatment. We can see that while (G1) is certainly true, (G2) is quite possibly false.

Powerful though Bill Gates is, there are some accidents that would remain true, whatever he wanted. Using the Gates antecedent allows to divide Σ into two sets: one includes the laws and other things that Gates is not powerful enough to change, the other set include the things he could change. We may be able to generalize the strategy that distinguishes between (G1) and (G2), by considering more powerful counterfactual antecedents in conditionals such as (G1) and (G2). On the other hand, we don't want the counterfactual supposition to be too powerful:

the transition elements are all non-conductors $\Box\!\!\rightarrow$ all bits of copper conduct electricity.

looks to be false.

Think of our conditionals as having the following form:

(T) C $\Box\!\!\rightarrow$ F

where 'C' denotes a counterfactual supposition and 'F' denotes an actual fact. The generalized Gates test suggests that (T) will always be true when F is a legal truth and C is an accident, but will sometimes be false when both are nomological facts or both are accidents. Lange's account of lawhood proposes that the laws are those that are stable under any counterfactual supposition that is an accident (or its negation).[2] As an account of law that looks to be problematic because it employs a concept 'accident' that is itself a concept of the kind we are wishing to elucidate: accidents are precisely the members of Σ that are not legal truths.

One of Lange's (many) smart moves is that he shows how to avoid this circularity. He does this by defining *sub-nomic stability*. Let Γ be a subset of Σ that includes all its consequences in Σ. Consider propositions of the form:

$p \,\Box\!\!\rightarrow m$

where p is the negation of some member of the complement of Γ in Σ and m is some member of Γ. The first pass at defining sub-nomic stability is to say that Γ is sub-nomically stable iff every proposition of this form is true. We can spell out sub-nomic stability this way. When Γ is sub-nomically stable, its members would still be true under every counterfactual supposition consistent with Γ. (We're still confining ourselves to propositions that are members of Σ, the sub-nomic truths, plus their negations.)

Lange argues that if g is an accident, than the only sub-nomically stable set containing g is Σ. Do the laws form a special sub-nomically stable set? Let Λ include all the laws, plus the metaphysical, mathematical, and logical necessities, but excluding any accidents. This set is sub-nomically stable for the reasons we've seen above. The members of Σ not in Λ are just the accidental truths. The counterfactual supposition that any of these is false, cannot change the laws, not the mathematical or logical truths etc. So we see that Λ is a non-trivially sub-nomically stable set. Does that suffice to pick out Λ? No, because there are other sub-nomically stable sets.

Take the mathematical and logical truths (but not the laws). There are stable under any supposition that an accident or law is false. So how do we pick out Λ? And furthermore, how do we pick out the laws from within Λ (in contrast to the logical and mathematical truths and so on)?

The next clever move from Lange is to show that the sub-nomically stable sets form a hierarchy. That is, for any pair of sub-nomically stable sets one is a subset of the other. We can thus pick out Λ as the largest sub-nomically stable set that is not Σ.

Lange's proof is important and it is worth seeing how it works. Consider a world in which there are two basic sub-nomic contingent facts, F and G. Trivially, the set {F, G} (i.e. Σ for this world), is sub-nomically stable. Could both {F} or {G} be sub-nomically stable? We shall see that one of other of {F} or {G} may be sub-nomically stable, but not both. Let us ask what would be the case were one of F or G false, i.e.:

$\neg F \vee \neg G \ \Box\!\!\rightarrow$?

Would F be false or would G be false? Or both? This is to ask: which of F or G is more *modally fragile*? In the actual world both F and G are true. We consider worlds at increasing distance from the actual world; eventually we will come to a world in which one or other of F and G is false. That is the nearest world in which $\neg F \vee \neg G$ is true. If F is false but G true in this world, then F is more modally fragile (less modally robust) than G. For example, note that it is true, F, that I am wearing red socks, and it is also true, G, that I am under 2.5m tall (in fact I am less than1.9m tall). Which is the nearest possible world in which one or other of these is false? Clearly it is a world in which I do not wear red sock, but in which I remain under 2.5m tall. In this case the following are true:

(i) $\neg F \vee \neg G \ \Box\!\!\rightarrow \neg F$
(ii) $\neg F \vee \neg G \ \Box\!\!\rightarrow G$.

F is more modally fragile than G.

Note that $\neg F \vee \neg G$ is consistent with F and is consistent with G. Let's now ask whether either of {F} or {G} is sub-nomically stable. (i) above tells us that {F} is not sub-nomically stable. If both {F} and {G} were to be sub-nomically stable, we would need:

(iii) $\neg F \vee \neg G \ \Box\!\!\rightarrow F$
(iv) $\neg F \vee \neg G \ \Box\!\!\rightarrow G$.

But clearly it cannot be that both (iii) and (iv) are true. It is this impossibility that means that at most one of {F} or {G} is sub-nomically stable. Lange's proof depends this principle, extending the case to any sets that are not related but the subset relation.

2 Lange on kinds

The fact proved by Lange, that the sub-nomically stable sets form a hierarchy ordered by the subset relation, means that we can order different kinds of (sub-nomic) proposition by their place in the hierarchy. So Σ, the set of all sub-nomic truths is, trivially, a sub-nomically stable set. So also is Λ, the sub-nomically stable set next largest in size to Σ. This contains the laws, but also the mathematical and logical truths and so forth. M, the set of metaphysical, mathematical, and logical truths also forms a sub-nomically stable set. The different levels in the hierarchy correspond to different grades of necessity. 'Natural necessity' is the grade possessed by all of Λ, 'metaphysical necessity' is the grade possessed by M, and so on. It may be that the laws properly speaking are the truths in Λ that are not in M. I.e. the laws of nature are those truths that are naturally necessary without being metaphysically necessary. Call the laws-properly-speaking 'Λ''.

More interestingly, within the laws, Λ', Lange claims there are divisions that correspond to different kinds of law with differing grades of natural necessity. For example, Lange thinks that there is a proper subset of Λ' that contains the fundamental dynamical laws, such as Newton's second law of motion and the force composition law (the 'parallelogram of forces'), but excludes the laws such as Coulomb's law and the law of gravitation which tell us what the particular forces are. Similarly he holds that the conservation laws have a special status that transcends the particular force laws.

Something similar seems to be true concerning the natural kinds of fundamental particle. These particles have certain basic, fundamental properties and certain derived properties. The basic and the derived properties are linked by natural law. Thus the muon has a certain spin, charge, and mass; these are its fundamental properties. Among its derived properties are is magnetic moment and half-life. The laws linking the fundamental and derived properties in some sense transcend the actual fundamental particles there are. Even if there had been other fundamental particles, these laws would still have held. That allows us to think counterfactually about what would have been the case had there been other particles. For example, Lange informs us, had there been a lepton less massive than the electron, then the electron would not have been stable, because there would have been some other particle into which it could decay. Let Ω be the true proposition that lists all the fundamental particle kinds there are and asserts that these are all the kinds of fundamental particle there are. Ω looks to be in Λ'—Ω would remain true whatever accidents happened to be otherwise. On the other hand, as we have just seen, the laws linking fundamental properties of the members of Ω to their derived properties are laws of a more robust sort, which would remain true even if Ω is false. This suggests that Ω possesses a low or event the lowest grade of natural necessity.

While I regard Lange's remarks on kinds stimulating and instructive, I think that their significance as an account of law and natural necessity remains up for discussion. First, the fact that:

> if *p* were not the case, *q* would still be the case

does not show that *q* has a higher grade of necessity than *p*. The truth of this conditional is consistent with the propositions just being independent. Lange doesn't suggest otherwise, but much of the discussion is framed, as above, in terms of one set of laws remaining as they are even if some other set were different. Consider the following claim made by Lange:

> (A) Had there been elementary particles not belonging to the actual kinds of quarks and leptons, then the same laws connecting the fundamental properties with the derivative properties would still have held; the force laws would have been no different.

He goes on to say:

> (B) Such subjunctive facts make stable a proper subset of Λ containing the laws connecting the fundamental properties with the derivative properties.

I want first to question whether (A) is true, secondly, whether (B) follows from (A), and thirdly whether (B) is as significant as Lange's presentation seems to suggest.

If (A) in fact true? One reason why it might not be is the following. We don't know yet what the fundamental laws of the universe are. Perhaps they form a very small, highly integrated set of laws, such that they together explain both the range of kinds of particle and the laws that connect the fundamental properties with the derivative properties. That is not implausible. The standard model was itself able to predict the existence of the W and Z bosons, the gluon, and the top and charm quarks, and their properties, indicating that Ω is not a fundamental fact, but a derived law. The force-mediating particles in particular are not independent of the force laws and other laws that govern the behaviour and nature of the fundamental particles. If one set of laws was responsible for both the existence of the particles and their natures (including their derived properties), then (A) would be false. Different fundamental particles would have required different fundamental laws, as a consequence of which the laws connecting the fundamental properties with the derivative properties would probably also be different.[3] Something similar may be said of other sets of kinds. There are only so many chemical elements and their isotopes. There are conceivable isotopes of elements that do not exist. The existing isotopes have various physical properties that derive from their structure. What explains why there are just these isotopes and not others? It is primarily the strong nuclear force. For example, there is no Helium-2 (diproton, an isotope with just two protons and no neutrons) because the actual value of the strength of the strong force prevents its existence. Had the strong force been 2% more powerful, then Helium-2 would have existed. It is also the strong force that explains the half-lives of the isotopes that do exist.[4] So statements such as:

> Had Helium-2 existed, the half-life of Polonium-210 would not have been any different.

are false. For Helium-2 to exist, the strong force would have to be stronger, and as a consequence the half-life of Polonium-210 would be different also.

One response to the above is to regard my argument against (A) as invalid, since it is implicitly appealing to a backtracking counterfactual. If I say 'if Lucy had not been at the party, I would have had a dull evening', that claim cannot be denied on the ground that in any (nearby) circumstances that Lucy did not go to the party she would have told me in advance and I would have not gone to the party, but would have done something else of interest. That reasoning is backtracking and deemed illegitimate by Lewis and others. Likewise the reasoning that if the kinds of particles were different, then the fundamental laws would be different too, seems to have a backtracking element to it, and ought to be outlawed also.

Should the argument against (A) be outlawed on backtracking grounds? First, the rejection of all backtracking reasoning and counterfactuals is contentious. Secondly, it is difficult to reject it in this case. Lewis avoids backtrackers by introducing small miracles—for example a change the firing of Lucy's neurons that causes her to not enter the party just before she arrives. This enables the antecedent of the counterfactual to hold, without changing the world's earlier history. That thus allows me to be at the party, unaccompanied and bored. The small miracle (which incidentally, makes some limited backtrackers true) occurs at a particular moment in time, too late for it to affect my attendance at the party. But what is the analogue to a small miracle in the case of (A)? The antecedent and consequent concern eternal truths, so it is difficult to see how anything like a small miracle could occur. An analogue would be something that allowed the antecedent to be true, i.e. for there to be different particle kinds, but without that happening as a result of any change to the fundamental laws. But on the supposition that Ω, the actual truth about the particle kinds, is a logical (or at least metaphysically necessary) consequence of the fundamental laws, nothing can fill that role. In short, small miracles may be able to break a chain of causal consequence but they cannot break a chain of logical consequence, and that is what would be required here. So I am not sure that the backtracking argument against (A) can be rejected on that ground. That argument is speculative, but it does suggest that we cannot take the truth of (A) for granted.

Note also that were the argument against (A) rejected on backtracking grounds, that would undermine either the inference from (A) to (B) or the significance of (B) itself. Let is say that a conditional:

> If *p* had not been the case, *q* would still have been the case

is held to be true on the grounds that its denial would require a backtracking argument. If so, it is difficult to see that the truth of the conditional shows that *q* is modally more robust than *p* and that *p* is modally more fragile. As pointed out above, the truth of that conditional is consistent with the truth of:

> If *q* had not been the case, *p* would still have been the case

Perhaps the denial of either counterfactual requires backtracking. The modal relative fragility of *p* and *q* is not decided by such conditionals. What we need is an answer to:

> If one or other of *p* or *q* were false, which would it be?

In our case, that is the question:

> If the particle kinds were different, or the (conjunctive) law relating the fundamental and derived properties of those kinds were different, which would it be?

It is not clear that the answer is that the particle kinds would be different (but the connecting law the same). And if we are allowed backtrackers, the answer might be that in any world where the one is different, the other is different also.

Note also that the exclusion of backtrackers, even temporal ones, threatens to undermine the significance of (B). Let **e** be the first event in the history of the universe, an event that is uncaused, unexplained etc. (perhaps the Big Bang is such, perhaps not). For any other event, **d**, in the history of the universe, the following is true:

> If **d** had not occurred, **e** would still have occurred

is true. As a result, if Λ is a sub-nomically stable set, then the set that is the union of Λ and {**e**} (and their consequences in Σ) is also sub-nomically stable. Thus **e** would have a grade of necessity not shared by any other event in history. While the idea that the initial conditions of the universe have a special status akin to that of the laws is not itself objectionable, it would be odd if that conclusion were reached simply on the basis of the rejection of backtracking counterfactuals.

Let us now look a little more at the idea of a sub-nomically stable set. According to Lange, the idea is revealing because, thanks to the proof sketched above, it sorts sub-nomic claims into a hierarchy and the different levels in this hierarchy correspond to different grades of necessity. Putting aside the caveats made above, let us concede that Ω, the statement about what particle kinds there are, is a subset of Λ, and so has some grade of natural necessity and is not an accident. On the other hand, if Lange is right, it has a very low grade of natural necessity, a lower grade than, for example, the symmetry and conservation laws. Is that fact revealing?

I want to suggest that it is perhaps less revealing than it might at first appear to be. If the hierarchy of sub-nomic sets sorted them into a smallish number of grades of necessity, corresponding to where we intuitively see potential differences of necessity (accidents, natural laws, metaphysical truths, logical truths), then that might well be significant. And if we found a small number of additional divisions, say sorting the laws into two or three different levels, then that would be an interesting discovery. On the other hand, if there are many more levels than we intuitively perceive, then one might find the hierarchy less significant. I will suggest that the latter might be the case.

The proof that sub-nomically stable sets form a hierarchy turned on the fact that the relation of 'being modally as robust or more robust than' provides a total order on all facts. Now let us suppose that it is also a well-ordering. Let Δ be a sub-nomically stable set. Let D be that member of Δ with the property that is it more modally fragile (less robust) than any other member of Δ. That is, the nearest possible world in which any of Δ is false, is a world where D is false but the remaining members of Δ are all true. Now consider Δ−D. Let E be any member of this set. D will be more modally fragile than E (by definition of D), so:

$$\neg D \vee \neg E \mathbin{\Box\!\!\rightarrow} \neg D$$
$$\neg D \vee \neg E \mathbin{\Box\!\!\rightarrow} E.$$

This shows that Δ−D is also sub-nomically stable. Now consider the least robust member, D′, of Δ−D. By the same reasoning, the set Δ−D−D′ is also sub-nomically stable. Repeating this construction leads to a series of sub-nomically stable sets, each one a subset of its predecessor formed by removing its most fragile member. It might seem that we can generalise this for cases which are not well-ordered by robustness (but are still totally ordered). Let λ mark some position in the ordering of facts by modal robustness and let Π_λ be the subset of the sub-nomically stable set Π formed by removing all facts that are at level λ or below. Then Π_λ looks as if it is a sub-nomically stable also. For every fact not in Π_λ is more modally fragile than every fact in Π. If that is correct, then there will not be just a few sub-nomically stable sets, but vast numbers. Correspondingly, grades of necessity will be multiplied to the same extent.

There is a fallacy in the preceding reasoning however. I have assumed that it if both A and B are more fragile than every member of some set Π, that is sufficient to show that we can draw a distinction between the modal grade of A and B on the one hand and the members of Π on the other. But matters are not so simple, because the following inference is invalid:

$$A \mathbin{\Box\!\!\rightarrow} C \wedge B \mathbin{\Box\!\!\rightarrow} C \textit{ therefore } A \wedge B \mathbin{\Box\!\!\rightarrow} C$$

For example I may suffer from an illness for which one pill of a powerful drug will cure me. But I know that taking two pills is an overdose that will kill me. I have two pills, A and B, in front of me. 'If I were to take pill A, I would recover' and 'If I were to take pill B, I would recover' are both true, but 'If I were to take pill A and to take pill B, I would recover' is false.

So although A might be highly modally fragile and B also, their conjunction might not be. More generally, from:

$$\forall q\, \forall p\, (q \text{ is atomic} \wedge q \text{ is consistent with } \Pi \wedge p \in \Pi) \rightarrow q \,\Box\!\!\rightarrow p$$

it does not follow that:

$$\forall q'\, \forall p\, (q' \text{ is compound} \wedge q' \text{ is consistent with } \Pi \wedge p \in \Pi) \rightarrow q' \,\Box\!\!\rightarrow p$$

That is, although every member p of Π is such that for every *atomic* proposition q consistent with Π, it is true that if q were the case, p would still be the case, it does not follow that the same is true for every *compound* proposition q'. As a consequence, sets of propositions of a range of modal fragility must be banded together to form a sub-nomically stable set. There isn't as much variation in grades of necessity as there is in modal robustness or fragility.

Nonetheless, that fact revealed in the preceding paragraph seems to be a consequence of the causal complexity of the actual world. Consider a much simpler world in which basic events happen at 1 second intervals, such that e_i causes e_{i+1} and so forth, governed by a simple causal law. In such a world it would seem that:

if e_m had not occurred e_n would still have occurred

is false if $m \leqslant n$ and true otherwise. In such a world later events are more fragile than earlier ones. Furthermore, let Σ be the set of all sub-nomic facts. Let Σ_i be the set formed from Σ by excluding all events after e_i. The sequence of sets $\{\ldots \Sigma_{i+1}, \Sigma_i, \Sigma_{i-1}, \ldots\}$ is a hierarchy of sub-nomically stable sets.

What are we to say then about laws concerning the kinds there are, such as Ω? Assume that Ω is a member of a sub-nomically stable set that is lower in the hierarchy than some other sub-nomically stable sets that include other, more robust laws (such as the symmetry laws). What does this show? It might just show that the different laws have different degrees of modal fragility. That in turn might reflect not a different grade of necessity but the logical structure of a law. It might seem that a force law, such as Coulomb's law, could easily fail to hold, for example in a world with a slightly different value of the permittivity of free space. Likewise laws concerning the range of kinds there are might, with relative ease, have been otherwise—as mentioned above, Helium-2 would exist in a world that (it seems) is only slightly different from ours. In these case there are similar, 'nearby' laws that would be true if the actual ones were not true. But for the structurally simpler symmetry laws, this is arguably not true.[5]

3 Conclusion

Marc Lange has given us an elegant account of what the laws are which brings with it the prospect of deep insights into the nature of necessity, showing how there are different grades of necessity. The laws are necessary, but less so than the metaphysical truths. Within the laws there are different grades of necessity. And it appears that laws stating which kinds there are have a lower grade of natural necessity than some of species of law—for example the apparently more general laws linking the fundamental properties of the kinds and their derived properties.

In this paper I have not rejected this view outright. But I have given some reasons to wonder whether it has been firmly established. First, one might wonder whether our intuitions concerning the relative modal robustness of the laws really reflect their natures. Or do they reflect our ignorance of the natures of the laws? It seems to me that there is ample room for the possibility that the very same laws that are responsible for the existence and non-existence of kinds are also the laws responsible for fixing the derived properties of those kinds. That is in fact the case for the existence of isotopes and their properties such as their half-lives, both of which are determined by the strong force. It seems at least plausible that the same is true for the fundamental particles.

Secondly, let us grant that it is true that were there different kinds of particle, the laws linking their fundamental and derived properties would be different. While suggestive, it is far from clear that this supports the idea that the kinds law belongs to a different sub-nomically stable set from the property laws.

Thirdly, we need to consider carefully what exactly the import of the hierarchy of sub-nomically stable sets. That hierarchy is closely related to the ranking of facts by their modal robustness. If the two were exactly the same, then the hierarchy of sub-nomically stable sets would be of little interest, and there would be infinitely many grades of necessity. But the two are not the same. The accidents, which vary regarding their modal robustness all have the same grade of necessity. On the other hand, this fact may reflect the causal complexity of the accidents in our world. In simpler worlds there are different grades of necessity among the accidents corresponding to their differing degrees of modal robustness. It is not clear to me that the same isn't true of the laws of the actual world.

Notes

[1] Note that for Lange, like Lewis and other regularity theorists, but unlike Armstrong, laws do not have a different logical form from other kinds of sub-nomic facts.

[2] I say 'or its negation' since I defined 'accident' to be factive, as a result of which the negations of accidents are not themselves accidents.

[3] I say 'probably' because a difference in fundamental laws does not logically necessitate a difference in all derivative laws. But the important ones will differ.

[4] In cases of fission and alpha-decay, not beta-decay, which is explained by the weak nuclear force.

[5] Compare this case. John is 210 cm tall. 'John is tall' is more robust than J̀ohn is 210cm ±5mm'.